SIAM J. APPLIED DYNAMICAL SYSTEMS
Vol. 4, No. 3, pp. 588–606

Spiral Waves in Nonlocal Equations*

Carlo R. Laing†

Abstract. [illegible]

Key words. [illegible]

AMS subject classifications. [illegible]

DOI. 10.11[illegible]

1. Introduction.

[illegible]

[20, 21]

2.1. Specific model.

$$[\ ^4 - \ ^2 + 1]\left(\frac{\partial u(\mathbf{x},t)}{\partial t} + u(\mathbf{x},t) + a(\mathbf{x},t)\right) = Bf\ u(\mathbf{x},t),\theta,\rho\ , \tag{4}$$

$$\tau\frac{\partial a(\mathbf{x},t)}{\partial t} = Au(\mathbf{x},t) - a(\mathbf{x},t), \tag{5}$$

$\mathbf{x}$ $\mathbb{R}^2$. (4)

$$\frac{\partial u(\mathbf{x},t)}{\partial t} = -u(\mathbf{x},t) + B\iint_{\Omega} w(|\mathbf{x}-\mathbf{y}|)f\ u(\mathbf{y},t),\theta,\rho\ d\mathbf{y} - a(\mathbf{x},t), \tag{6}$$

Ω $\mathbb{R}^2$

$$w(r) = \int_0^{\infty}\frac{sJ_0(rs)}{s^4+s^2+1}\,ds, \tag{7}$$

J_0 (6),

$$F\ u_t + u + a\ = F\ w\ F\ Bf\ u,\theta,\rho\ , \tag{8}$$

F

$$F\ w\ = \frac{1}{s^4+s^2+1}, \tag{9}$$

s ( ) $F\ w$ (4). w (9), ( ), 1 20, 21 (5) (6) 11, 12, f

$$f\ u,\theta,\rho\ = H(u-\theta)e^{-\rho/(u-\theta)^2},$$

H $\rho \to 0,\ f(u,\theta,\rho) \to H(u-\theta).$ Ω R

$$\left.\frac{\partial u}{\partial r}\right|_{r=R} = \left.\frac{\partial^3 u}{\partial r^3}\right|_{r=R} = 0.$$

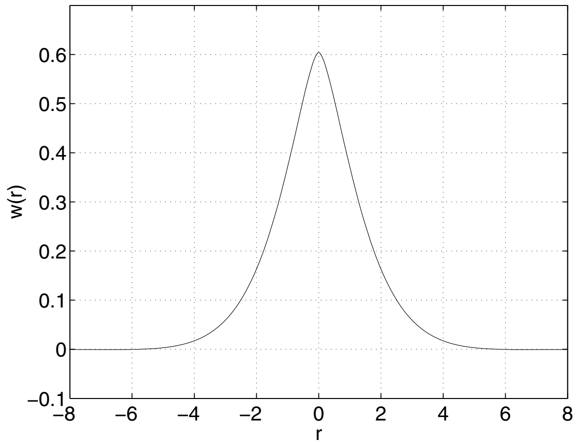


Figure 1. *The coupling function* $w(r)$, *given by* ( ).

$\partial/\partial t$ (4) (5) $-\omega \times \partial/\partial\theta$, ω 2, .

$$(10)\qquad \left[\ {}^4 - {}^2 + 1\right]\left(-\omega\frac{\partial u}{\partial \theta} + u + a\right) = B f\ u, \theta, \rho\ ,$$

$$(11)\qquad -\omega\tau\frac{\partial a}{\partial \theta} = Au - a.$$

(11)

$$(12)\qquad a = A\left(I - \omega\tau\frac{\partial}{\partial\theta}\right)^{-1} u,$$

I , a u. (10), u

$$(13)\qquad \left[\ {}^4 - {}^2 + 1\right]\left[I - \omega\frac{\partial}{\partial\theta} + A\left(I - \omega\tau\frac{\partial}{\partial\theta}\right)^{-1}\right] u = B f\ u, \theta, \rho\ .$$

(13), . (13) 2, . ω (13) (u, ω) . (13) (4) (5) , .

$(A = 2, B = 0.5, \rho = 0.1, \theta = 0.2)$, (4) (5) (

$(A+1)u = Bf(u,\theta,\rho)$) $u \approx 0, 0.5$ 1. [illegible] (4) (5). [illegible] 1 [illegible] (4) (5).

2.2. Stability. [illegible] (4) (5)

$$\frac{\partial u}{\partial t} = -u + B \quad 4$$

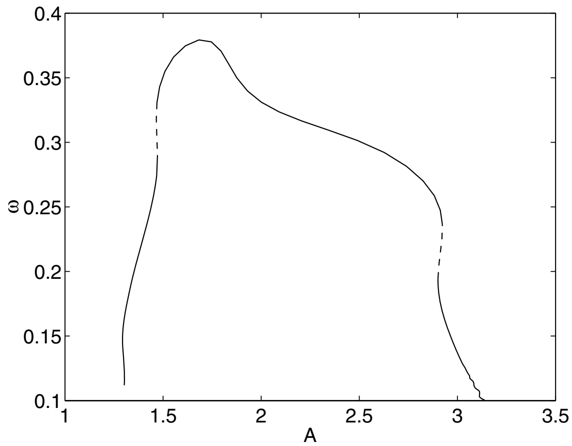


Figure 2. *Rotational speed ω as a function of adaptation strength A. Solid lines indicate stable solutions, while dashed indicate unstable. See text for other parameters. Clicking on the above image displays the associated movie showing spirals at di erent points on the curve (1 _01. ().*

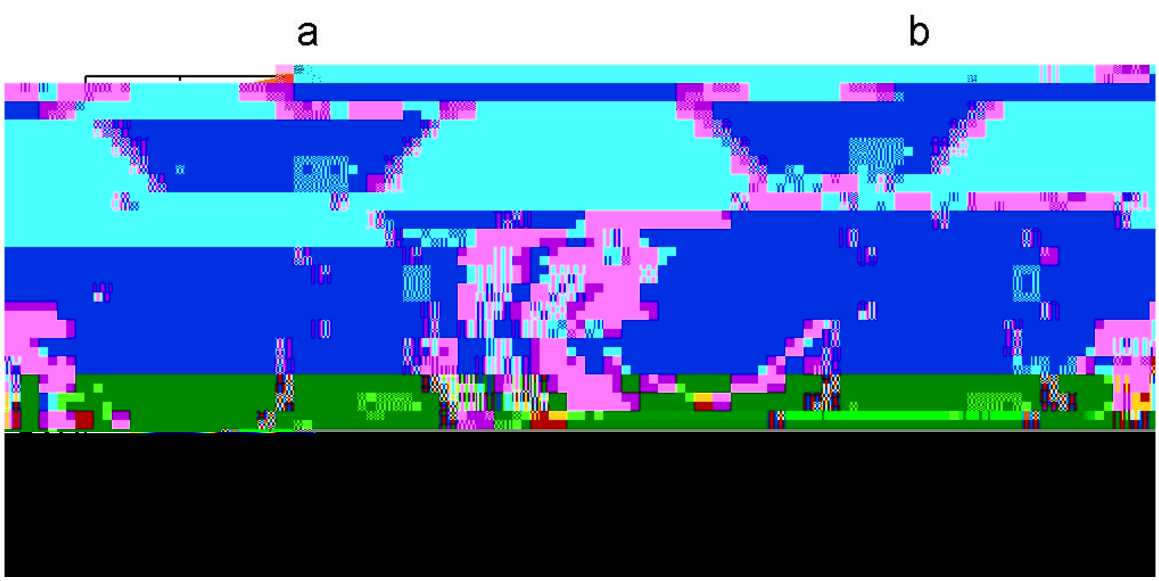
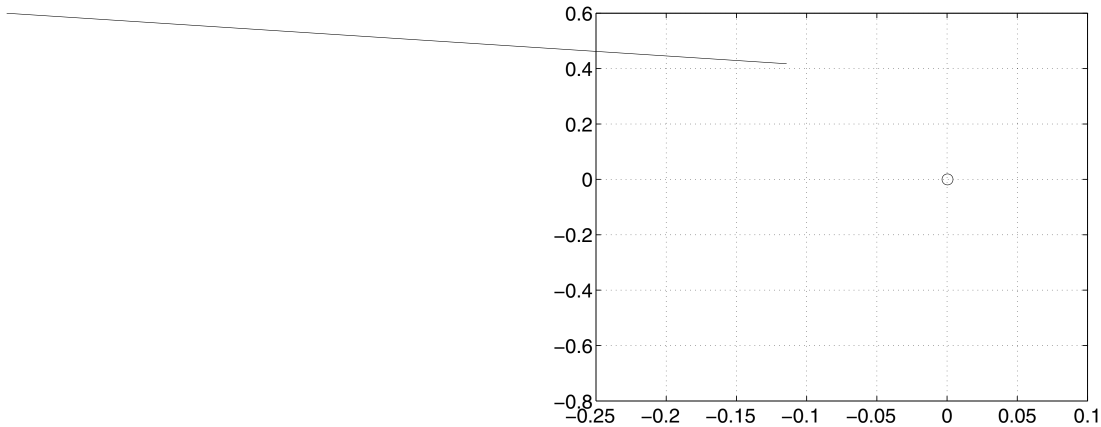
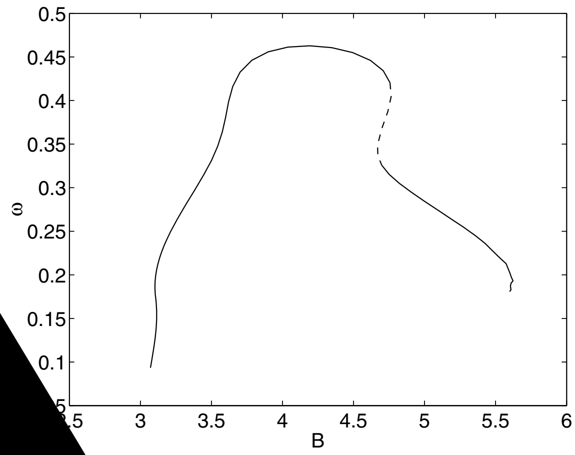
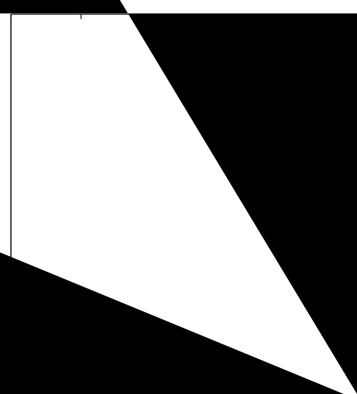


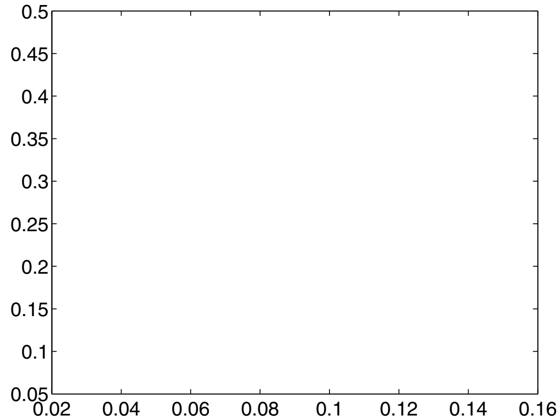
Figure 3.


0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−0.25
−0.2
−0.15
−0.1
−0.05
0
0.05
0.1



[redacted] function of B, the strength of the nonlinear term. Solid lines indicate [redacted]nstable. See text for other parameters. Clicking on the above image [redacted]als at di erent points on the curve (1 0 . ().




0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16

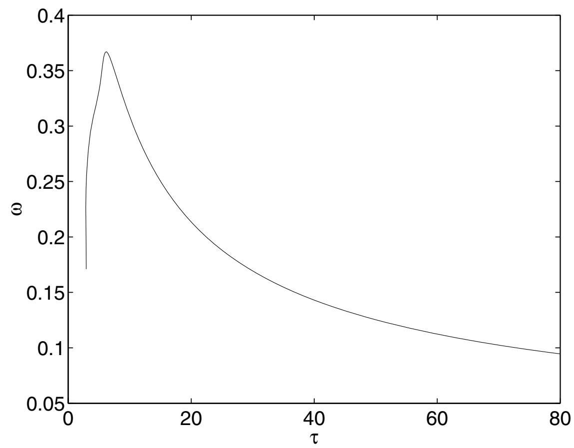
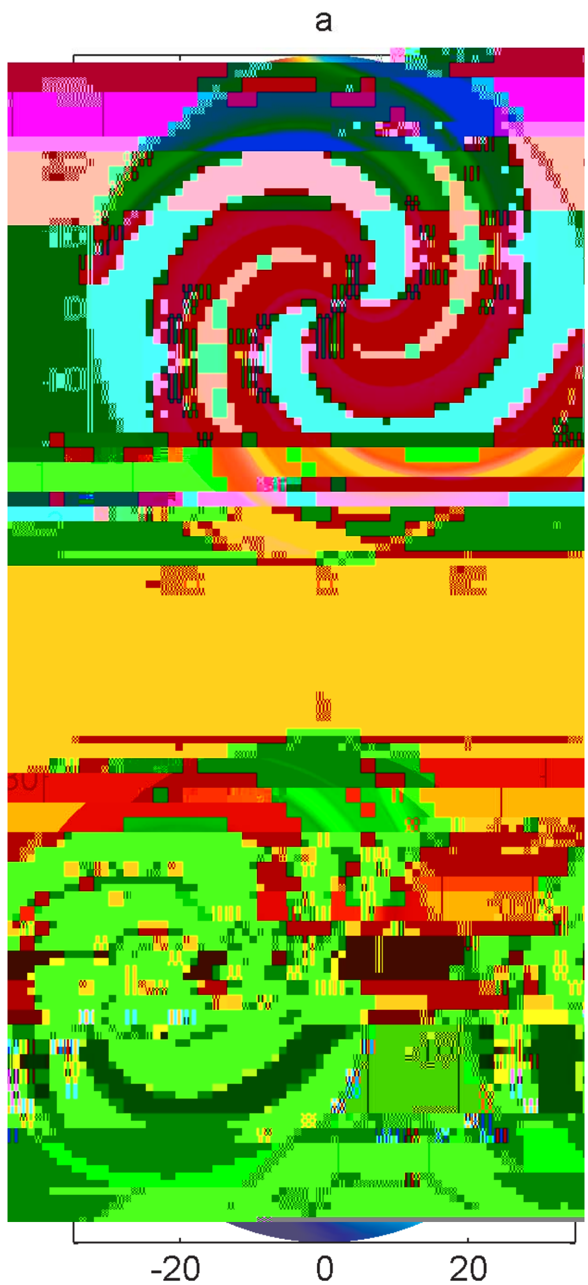


Figure 10. *Rotational speed ω as a function of τ. See text for other parameters. Clicking on the above image displays the associated movie showing spirals at di erent points on the curve* (1 _0 . ().


a
-20
0
20

[illegible]

4. Conclusion.

[illegible]

M. A. Dahlem and S. C. Müller, *Self-induced splitting of spiral-shaped spreading depression waves in chicken retina*, .] ., 11 (1), . 1 .

B. Ermentrout, *Neural networks as spatio-temporal pattern-forming systems*, . (. ., 1 (1), . 0.

10 C. Fohlmeister, W. Gerstner, R. Ritz, and J. L. van Hemmen, *Spontaneous excitations in the visual cortex: Stripes, spirals, rings, and collective bursts*,] ., (1), . 0 1 .

11 S. E. Folias and P. C. Bressloff, *Breathing pulses in an excitatory neural network*, . . ., (00), . 0 .

1 L. Glass and M. C. Mackey, *From Clocks to Chaos: The Rhythms of Life*

A. Scheel, *Bifurcation to spiral waves in reaction-di usion systems*, .] .]., (1), .1 1 1 .

J. A. Sepulchre and A. Babloyantz, *Motions of spiral waves in oscillatory media and in the presence of obstacles*, . . , (1), .1 1 .

P. Tass, *Oscillatory cortical activity during visual hallucinations*, . . ., (1), . 1 .

0 B. Vasiev, F. Siegert, and C. Weijer, *Multiarmed spirals in excitable media*, . . ., (1), . .

1 A. Winfree, *Are cardiac waves relevant to epileptic wave propagation?*,]]]]], .] . (, ., (-] (, , 00 .

R. M. Zaritski and A. M. Pertsov, *Stable spiral structures and their interaction in two-dimensional excitable media*, . . ( ), (00), 0 1 0.

V. S. Zykov and H. Engel, *Dynamics of spiral waves under global feedback in excitable domains of di erent shapes*, . . ( ), 0 (00), 01 01.

V. S. Zykov, A. S. Mikhailov, and S. C. Müller, *Controlling spiral waves in confined geometries by global feedback*, . . ., (1), . 01.