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## **SPIRAL WAVES IN NONLOCAL EQUATIONS** 589

<span id="page-3-0"></span>
$$
\sum_{i=1}^{n} k_i = \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}
$$

(4) 
$$
\begin{bmatrix} 4 & -2 & +1 \end{bmatrix} \left( \frac{u(\mathbf{x}, t)}{t} + u(\mathbf{x}, t) + a(\mathbf{x}, t) \right) = Bf u(\mathbf{x}, t), \quad (5)
$$

$$
\frac{a(\mathbf{x}, t)}{t} = A u(\mathbf{x}, t) - a(\mathbf{x}, t),
$$

**4.** 
$$
\mathbf{x} \quad \mathbb{R}^2
$$
.  $\qquad \qquad$  (4)  $\mathbf{f} \qquad \qquad$   $\qquad \qquad \qquad$   $\qquad \qquad \qquad$   $\qquad \qquad \$ 

$$
\mathbf{k} \cdot \quad \Omega \quad \mathbb{R}^2 \quad \mathbf{k} \cdot \quad \mathbf{f} \quad \mathbf{v} \quad \mathbf{f} \quad \mathbf{v} \quad
$$

$L_{t}$	$J_{0}$	$L_{t}$	$f$																																																																							
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**44.** 
$$
F \cdot \frac{1}{t}
$$
 **46.**  $F \cdot \frac{1}{s^4 + s^2 + 1}$ 

$$
\mathbf{L}_{1} \mathbf{S} = \mathbf{I}_{1} \mathbf{F} \mathbf{W} \mathbf{A} + \mathbf{L}_{2} \mathbf{F} \mathbf{A} + \mathbf{L}_{3} \mathbf{F} \mathbf{A} + \mathbf{L}_{4} \mathbf{F} \mathbf{A} + \mathbf{L}_{5} \mathbf{F} \mathbf{A} + \mathbf{L}_{6} \mathbf{F} \mathbf{A} + \mathbf{L}_{7} \mathbf{F} \mathbf{A} + \mathbf{L}_{8} \mathbf{F} \mathbf{A} + \mathbf{L}_{9} \mathbf{F} \mathbf{A} + \mathbf{L}_{1} \mathbf{F} \mathbf{A} + \mathbf{L}_{1} \mathbf{F} \mathbf{A} + \mathbf{L}_{2} \mathbf{F} \mathbf{A} + \mathbf{L}_{3} \mathbf{F} \mathbf{A} + \mathbf{L}_{4} \mathbf{F} \mathbf{A} + \mathbf{L}_{5} \mathbf{F} \mathbf{A} + \mathbf{L}_{6} \mathbf{F} \mathbf{A} + \mathbf{L}_{7} \mathbf{F} \mathbf{A} + \mathbf{L}_{8} \mathbf{F} \mathbf{A} + \mathbf{L}_{9} \mathbf{F} \mathbf{A} + \mathbf{L}_{1} \mathbf{F} \mathbf{A} + \mathbf{L}_{1} \mathbf{F} \mathbf{A} + \mathbf{L}_{2} \mathbf{F} \mathbf{A} + \mathbf{L}_{3} \mathbf{F} \mathbf{A} + \mathbf{L}_{5} \mathbf{F} \mathbf{A} + \mathbf{L}_{6} \mathbf{F} \mathbf{A} + \mathbf{L}_{7} \mathbf{F} \mathbf{A} + \mathbf{L}_{8} \mathbf{F} \mathbf{A} + \mathbf{L}_{9} \mathbf{F} \mathbf{A} + \mathbf{L}_{1} \mathbf{F} \mathbf{A} + \mathbf{L}_{1} \mathbf{F} \mathbf{A} + \mathbf{L}_{2} \mathbf{F} \mathbf{A} + \mathbf{L}_{3} \mathbf{F} \mathbf{A} + \mathbf{L}_{5} \mathbf{F} \mathbf{A} + \mathbf{L}_{6} \mathbf{F} \mathbf{A} + \mathbf{L}_{7} \mathbf{F} \mathbf{A} + \mathbf{
$$

$$
\mathbf{a} \quad H \quad \mathbf{b} \quad = \quad \Omega \quad \mathbf{b} \quad \mathbf{b} \quad = \quad \frac{1}{r} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1
$$

<span id="page-4-0"></span>

**Figure 1.** The coupling function  $w(r)$  $w(r)$  $w(r)$ , given by  $( )$ .

We look for spiral waves that undergo rigid rotation about the center of the disk. For these solutions, we can replace ∂/∂t in [\(4\)](#page-3-0)–[\(5\)](#page-3-0) with − × ∂/∂θ, where is the angular velocity of the spiral [\[2,](#page-16-0) [3\]](#page-16-0). This results in the equations <sup>4</sup> − ∇<sup>2</sup> + 1 <sup>−</sup> ∂u ∂θ <sup>+</sup> <sup>u</sup> <sup>+</sup> <sup>a</sup> (10) = Bf u, θ, ρ , <sup>−</sup>ωτ ∂a ∂θ (11) <sup>=</sup> Au <sup>−</sup> a.

(12) 
$$
(11)
$$
  $a = A(1 - -1)^{-1}u$ 

$$
\begin{array}{c}\n\mathbf{h}_{1} & I \\
\mathbf{v}_{2} & \mathbf{v}_{1} \\
\mathbf{v}_{2} & \mathbf{v}_{2} \\
\mathbf{v}_{3} & \mathbf{v}_{3} \\
\mathbf{v}_{4} & \mathbf{v}_{2} \\
\mathbf{v}_{5} & \mathbf{v}_{3} \\
\mathbf{v}_{6} & \mathbf{v}_{7} \\
\mathbf{v}_{7} & \mathbf{v}_{8} \\
\mathbf{v}_{8} & \mathbf{v}_{9} \\
\mathbf{v}_{9} & \mathbf{v}_{1} \\
\mathbf{v}_{1} & \mathbf{v}_{1} \\
\mathbf{v}_{1} & \mathbf{v}_{2} \\
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\mathbf{v}_{2} & \mathbf{v}_{3} \\
\mathbf{v}_{3} & \mathbf{v}_{3} \\
\mathbf{v}_{4} & \mathbf{v}_{2} \\
\mathbf{v}_{5} & \mathbf{v}_{3} \\
\mathbf{v}_{6} & \mathbf{v}_{7} \\
\mathbf{v}_{7} & \mathbf{v}_{8} \\
\mathbf{v}_{9} & \mathbf{v}_{1} \\
\mathbf{v}_{1} & \mathbf{v}_{1} \\
\mathbf{v}_{1} & \mathbf{v}_{2} \\
\mathbf{v}_{2} & \mathbf{v}_{3} \\
\mathbf{v}_{3} & \mathbf{v}_{1} \\
\mathbf
$$

 $\mathbb{R}^n$  of the rotational symmetry of the problem, the problem  $f(1^g)$ , each member of the family by another intervals obtained from another in the family by an angular rotation. This degeneracy can be eliminated by another eliminated by another equation which pins the pins  $p_1$ ,  $p_2$ ,  $p_3$ . The inclusion of  $p_4$  inclusion of the inclusion of the inclusion allows us to the inclusion of  $f = \frac{1}{2}$  in  $f = \frac{1}{2}$  in  $f = \frac$  $\overline{u}$   $\overline{$ In section  $\mathcal{I}$  we will find one solution of  $\mathbf{f}(1\mathbf{V})$  by direct numerical integration of  $\mathbf{f}(4)(5)$  $\mathbf{f}(4)(5)$  $\mathbf{f}(4)(5)$ then numerically continue this solution as a parameter is varied, generating a parameter is varied, and  $\alpha$ family of solutions of  $\mathbf f$  $(A = 2, B = 3.5, = 0.1, = 0.2),$  the system use  $(A = 2, B = 3.5, = 0.1, = 0.2),$  the the [\(4\)](#page-3-0)–[\(5\)](#page-3-0) has the fixed points in the absence of spatial coupling  $f(x) = \frac{1}{2} \int_{0}^{x} f(x) \, dx$ 

<span id="page-5-0"></span>





**Figure 2.** Rotational speed ω as a function of adaptation strength A. Solid lines indicate stable solutions, while dashed indicate unstable. See text for other parameters. Clicking on the above image displays the associated movie showing spirals at dievent points on the curve  $(14 \quad 01.$  ().



**Figure 3.**





function of B, the strength of the nonlinear term. Solid lines indicate nstable. See text for other parameters. Clicking on the above image als at di-erent points on the curve (14  $\sim$  0  $\sim$   $\frac{1}{2}$ ).





**Figure 10.** Rotational speed ω as a function of τ . See text for other parameters. Clicking on the above image displays the associated movie showing spirals at different points on the curve ( $14\sqrt{0}$ . (i).



 $\mathsf{a}$ 



<span id="page-16-0"></span>

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