



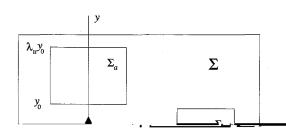
Successive homoclinic tangencies to a limit cycle

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	Abstract
	The dynamics near a perturbed degenerate homoclinic connection to a periodic orbit in three dimensions is modeled by
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2.1. Boundary conditions

An important feature of the definition of the return sections Σ_a and Σ_b is that the linear map L takes the lower boundary $y = y_0$ of Σ_a to its upper boundary $y = \lambda_u y_0$, while L takes the right boundary $x = x_0$ of Σ_b onto its left boundary $x = \lambda_c x_0$. Thus a point $A = x_0$

	and minimum on the interval $[y_0, \lambda_u y_0]$, as for example,	$\lambda_u y_0 \frac{\lambda_s \overline{x}}{}$	
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4. Simple fixed points

The simplest periodic orbits of the flow are the trajectories which link up with themselves after only one pass through the global region of the flow. These correspond to fixed points $(x,y) \in \Sigma_a$ of our mapping which satisfy $(x,y) = L^m G(x,y)$, or more explicitly

$$x = \lambda_s^m \phi(y) ,$$

$$y = \lambda_u^m \left\{ \left[\mu + \gamma \bar{x}(x, y) \right] y / y_0 + \epsilon f(y) \right\} .$$
 (20)

The substitution of the first equation $x = \lambda_s^m \phi(y)$ into

fixed points merge again in a second saddle-node tangency and disappear. Thus as μ varies from positive to negative, two cycles are created in a saddle-node bifurcation and then the same two cycles merge and are destroyed in another saddle-node bifurcation.

Now we find where the saddle-node bifurcations occur relative to the primary homoclinic tangencies. The saddle nodes occur when

$$\beta = -\epsilon f'(y) \,, \tag{24}$$

and a simple computation shows that the corresponding fixed point $(\lambda_n^m \phi(v), v)$ is also given by a solu-

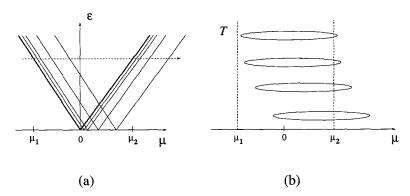
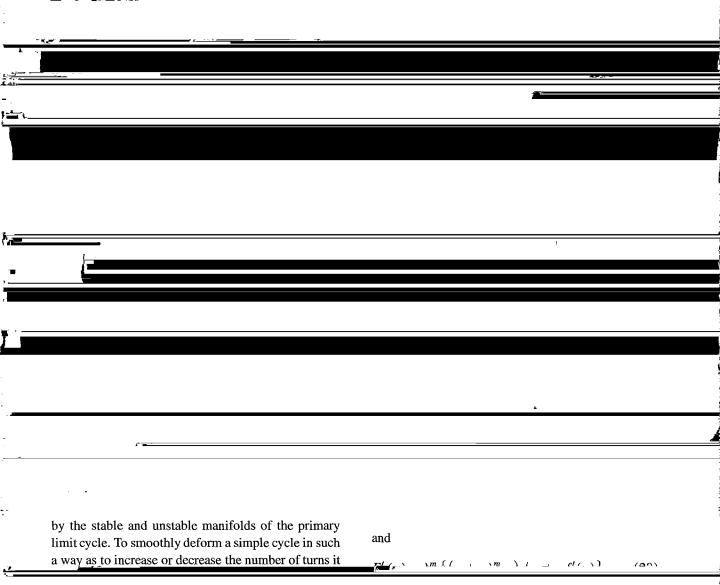
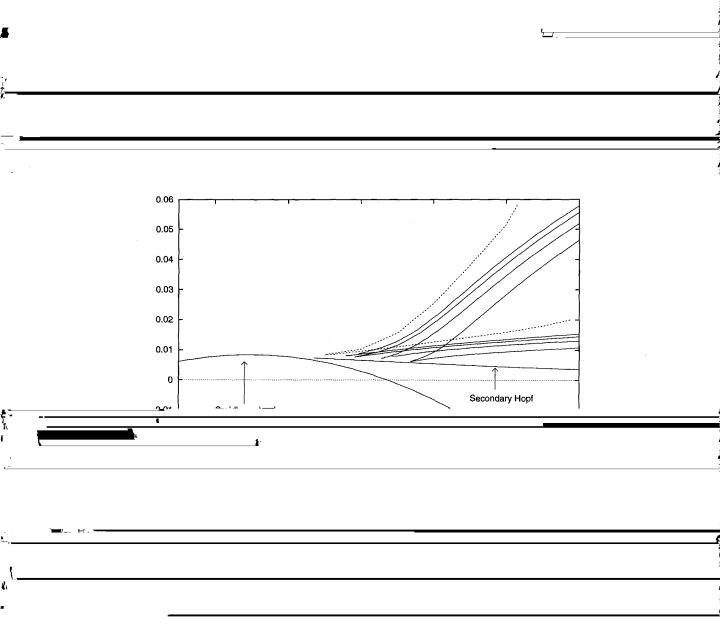


Fig. 6. Shown in (a) are curves of saddle-node bifurcations in the (μ, ϵ) parameter space which converge upon the leading tangency curve (bold curve, $\mu > 0$) and the trailing tangency curve (bold curve, $\mu < 0$). Shown in (b) is a sketch of period T versus μ for the limit cycles along a parameter path with fixed ϵ (dashed line shown in (a)). The leading tangency occurs at μ_2 , while trailing occurs at μ_2 For both (a) and (b) λ , $\lambda_2 < 1$





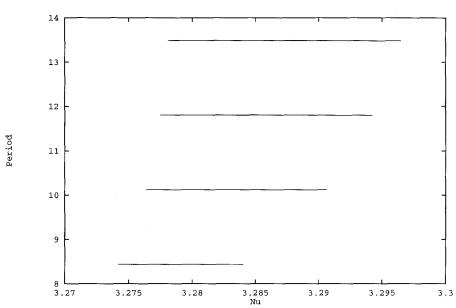


Fig. 10. Plot of period versus ν at $\eta = -5.5$ for periodic orbits corresponding to tongues with rotation numbers 1/5, 1/6, 1/7 and 1/8, for Fig. (34) with parameters as in Fig. 8. (The 1/5 tongue has the largest radial the 1/9 the birth of NR.

of a flow and our model map, a plot of orbit period versus a parameter path through the two tangencies yields closed bubbles. is gratefully acknowledged. C.L. would like to thank the Department of Physics, University of Auckland, for financial assistance to attend the School. The au-

other example where two tangency curves come together in parameter space and the associated saddle-node bifurcations at the separate tangencies may be linked together. Period versus parameter plots in this situation also show the accumulation of saddle-node bifurcations on homoclinic tangencies, but in this case the orbits are linked by a single curve which zigzags in

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References

[1] S.N. Chow, J.K. Hale and J. Mallet-Paret, An example of hiturcation_to_homoclinic orbits_J. Diff. Eans. 37 (1983)