PHYSICA D

Successive homoclinic tangencies to a limit cycle

P. Hirschberg [a], Carlo Laing [b,1]

[a] *Department of Physics, University of California at Berkeley, Berkeley, CA 94720, USA*

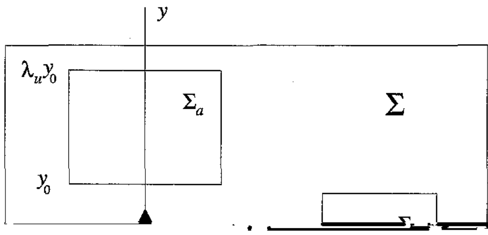
[b] *Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand*

Received 4 November 1994; revised 26 May 1995; accepted 26 May 1995

Communicated by J.D. Meiss

Abstract

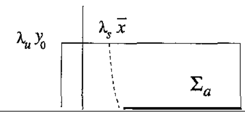
The dynamics near a perturbed degenerate homoclinic connection to a periodic orbit in three dimensions is modeled by



2.1. Boundary conditions

An important feature of the definition of the return sections Σ_a and Σ_b is that the linear map L takes the lower boundary $y = y_0$ of Σ_a to its upper boundary $y = \lambda_u y_0$, while L takes the right boundary $x = x_0$ of Σ_b onto its left boundary $x = \lambda_s x_0$. Thus a point $A =$

and minimum on the interval $[y_0, \lambda_u y_0]$, as for example,



4. Simple fixed points

The simplest periodic orbits of the flow are the trajectories which link up with themselves after only one pass through the global region of the flow. These correspond to fixed points $(x,y) \in \Sigma_a$ of our mapping which satisfy $(x,y) = L^m G(x,y)$, or more explicitly

$$x = \lambda_s^m \phi(y)\,,$$
$$y = \lambda_u^m \left\{ \left[\mu + \gamma \bar{x}(x,y)\right] y/y_0 + \epsilon f(y) \right\}\,. \qquad (20)$$

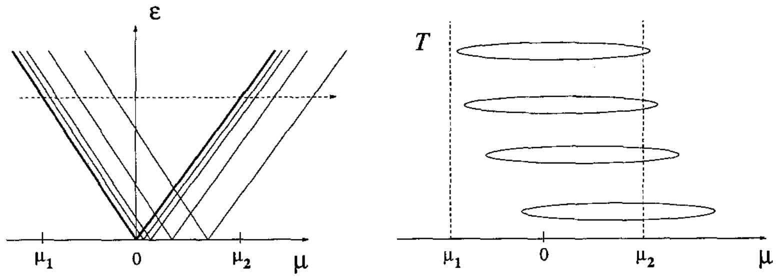
The substitution of the first equation $x = \lambda_s^m \phi(y)$ into the second equation yields an equation for y decoupled

fixed points merge again in a second saddle-node tangency and disappear. Thus as μ varies from positive to negative, two cycles are created in a saddle-node bifurcation and then the same two cycles merge and are destroyed in another saddle-node bifurcation.

Now we find where the saddle-node bifurcations occur relative to the primary homoclinic tangencies. The saddle nodes occur when

$$\beta = -\epsilon f'(y)\,, \qquad (24)$$

and a simple computation shows that the corresponding fixed point $(\lambda_s^m \phi(y), y)$ is also given by a solu-



(a) (b)

Fig. 6. Shown in (a) are curves of saddle-node bifurcations in the (μ, ϵ) parameter space which converge upon the leading tangency curve (bold curve, $\mu > 0$) and the trailing tangency curve (bold curve, $\mu < 0$). Shown in (b) is a sketch of period T versus μ for the limit cycles along a parameter path with fixed ϵ (dashed line shown in (a)). The leading tangency occurs at μ_2, while trailing occurs at μ_1. For both (a) and (b) $\lambda\lambda < 1$

by the stable and unstable manifolds of the primary limit cycle. To smoothly deform a simple cycle in such a way as to increase or decrease the number of turns it

and

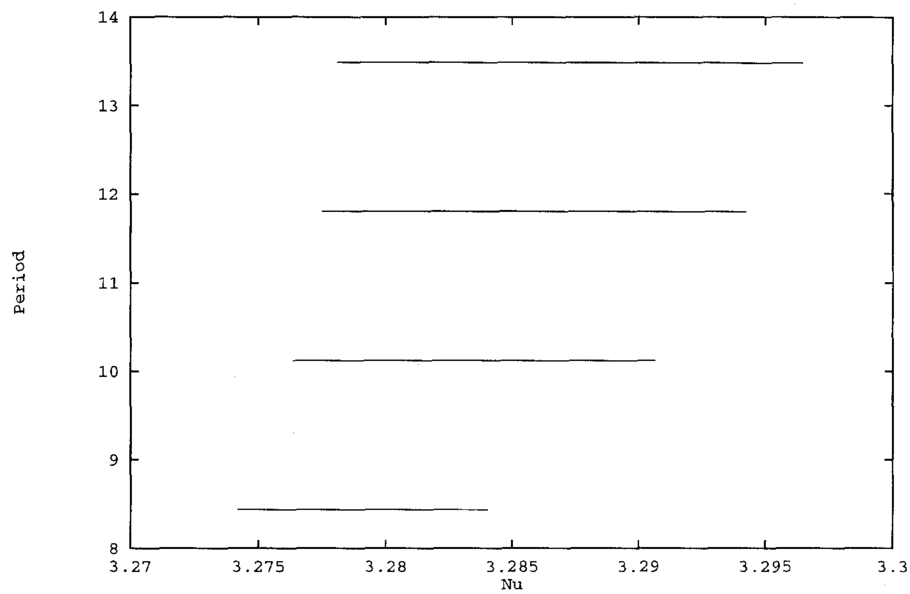


Fig. 10. Plot of period versus ν at $\eta = -5.5$ for periodic orbits corresponding to tongues with rotation numbers 1/5, 1/6, 1/7 and 1/8, [redacted]